Model Reference Adaptive Control of Large Structural Systems

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Model reference adaptive control procedures that do not require explicit parameter identification are considered for large structural systems. Although such applications have been shown to be feasible for multivariable systems, provided there exists a feedback gain matrix which makes the resulting input/output transfer function strictly positive real, it is now shown that this constraint is overly restrictive and that only positive realness is required. Subsequent consideration of a simply supported beam shows that if actuators and sensors are collocated, then the positive realness constraint will be satisfied and the model reference adaptive control will then indeed be suitable for velocity following when only velocity sensors are available and for both position and velocity following when velocity plus scaled position outputs are measured. In both cases, all states, regardless of system dimension, are guaranteed to be stable.

I. Introduction

THE need for parameter estimation and/or adaptive control of any system arises because of ignorance of the system's internal structure and critical parameter values, as well as changing control regimes. A large structural system (LSS) is substantially more susceptible to these problems. The most crucial problem of adaptive control of large structures is that the plant is very large or infinite-dimensional and, consequently, the adaptive controller must be based on a low-order model of the system in order to be implemented with an on-line/onboard computer. However, any controller based on a reduced-order model (ROM) must operate in closed loop with the actual system; thus it interacts not only with the ROM but also with the residual subsystem (through the spillover and model error terms).

One particular adaptive algorithm that seems applicable to LSS is the direct (or implicit) model reference-based approach taken by Sobel et al.[1,2] In particular, using command generator tracker (CGT) theory,[3] with Lyapunov stability-based design procedures, they were able to develop for step commands a model reference adaptive control (MRAC) algorithm that, without the need for parameter identification, forced the error between plant and model (which need not be of the same order as the plant) to approach zero, provided that certain plant/model structural conditions are satisfied.

Such an adaptation algorithm is very attractive for the control of large structural systems since it eliminates the need for explicitly identifying the large number of modes that must be modeled, and, furthermore, eliminates the spillover effects.

Relative to the conditions that must be satisfied, it was shown that asymptotic stability results provided that the plant input/output transfer matrix is strictly positive real for some feedback gain matrix and provided that there exists a bounded solution to the corresponding deterministic CGT problem.

Such a solution, however, does not always exist for structural problems with velocity sensors and, furthermore, the transfer matrix for structural systems is positive real (not strictly positive real) for collocated actuators and rate sensors.[4]

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Thus, with the objective of increasing the applicability of the MRAC algorithm of Refs. 1 and 2, this paper shows that output following still results when the original strictly positive real constraint is relaxed to a positive real constraint.

The net result is a control suitable for high-order structures which a priori requires only an arbitrary initial gain matrix and a set of non-negative parameters for the adaptive tuning logic. The true system order need not be known a priori.

Results show that velocity following is possible when only velocity sensors are used, and that both velocity and position following is possible when a combined velocity and position output is used.

To this effect, Sec. II discusses the problem statement of the pertinent adaptive control concepts, while Sec. III analyzes the specific beam control application. Finally, results are presented in Sec. IV, and conclusions and recommendations are given in Sec. V.

II. Adaptive Control of Large-Scale Structural Systems

A. The Problem Setting

We consider *linear, time-invariant* stable large-scale structural systems (LSS) of the following form:

$$\frac{\partial v}{\partial t} = Av + Bf \qquad v(0) = v_0 \tag{1}$$

$$y = Cv \tag{2}$$

where the state v is in some state space H, and the control vector f and observation vector y both have the dimension M [which denotes the number of (independent) actuators and/or sensors]. For large structural systems (LSS) the dimension H is less than infinity, but still very large, and the operators A, B, and C are all matrices with appropriate dimensions.

B. Reduced-Order Modeling

In order to define a model suitable for control purposes, we will decompose the large-dimensional state vector into a (small-dimensional) nominal component and a (large-dimensional) residual component. To this effect, let H_N and H_R be subspaces of the total state space H with dimension $H_N = N$ and dimension $H_R = R$, and $H = H_N + H_R$. Define the projection operators P_N and P_R (not necessarily orthogonal) and let $v_N = P_N v$ and $v_R = P_R v$. This decomposes v into $v = v_N + v_R$ and the system of Eq. (1) into

$$\dot{v}_N = A_N v_N + A_{NR} v_R + B_N f, \quad v_N(0) = P_N v_0 \tag{3}$$

$$\dot{v}_R = A_{RN} v_N + A_R v_R + B_R f, \quad v_R(0) = P_R v_0 \tag{4}$$

$$y = C_N v_N + C_R v_R \tag{5}$$

Concatenation of the v_N and v_R components into the vector $v = (v_N^T, v_R^T)^T$ yields:

$$\dot{v} = Av + Bf, \qquad y = Cv \tag{6}$$

where

$$A = \begin{bmatrix} A_N & A_{NR} \\ A_{RN} & A_R \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} B_N \\ B_R \end{bmatrix}$$

The dimension of v $(N+R)$ will be denoted as N_p in the sequel.

C. Adaptive Control System Design

1. Formulation

The continuous linear model reference control problem is to be solved for the linear process equations defined by Eqs. (3-5). The range of the process parameters is assumed to be bounded and all possible (A,B) pairs [see Eq. (1)] are assumed controllable and output stabilizable. The objective is to find, without explicit knowledge of A and B, the control f such that the plant output vector y approximates "reasonably well" the output of the following model forced by a step command:

$$\dot{v}_m(t) = A_m v_m(t) + B_m f_m(t) \tag{7}$$

$$y_m(t) = C_m v_m(t) \tag{8}$$

where $v_m(t)$ is the model state vector $(N_m \times 1)$, $f_m(t)$ is the model step input or command $(M \times 1)$, $y_m(t)$ is the model output vector $(M \times 1)$, and A_m, B_m are matrices with the appropriate dimensions.

As a basis for generating the defining matrices (A_m, B_m, C_m) for this reference model, one might consider the selection of v_m and f_m as having the same dimensions as v_N and f, respectively. Ignoring v_R, the model matrices would then be selected to represent desired values for A_N, B_N, C_N (e.g., with desirable damping).

Furthermore, the controller structure is to be such that as time approaches infinity the error $(y - y_m)$ approaches zero. This is to be valid regardless of model structural characteristics, i.e., even if the so-called conditions for perfect model following do not hold.

To facilitate the controller development, it is useful to incorporate the command generator tracker concept developed by Broussard.[3] When $y = y_m$ for $t \geq 0$, i.e., perfect tracking occurs, the corresponding plant state and control trajectories will be denoted as $v^*(t)$ and f^*, respectively. By definition then the ideal plant response v^* is such that

$$y^* = C_N v_N^* + C_R v_R^* = C_m v_m = Cv^* \tag{9}$$

and

$$\dot{v}^* = Av^* + Bf^* \tag{10}$$

Furthermore, the asterisked quantities will be assumed to be linearly related to the model command f_m and model state x_m. More precisely, matrices S_{ij} must exist such that

$$v^* = S_{11} v_m + S_{12} f_m \tag{11}$$

$$f^* = S_{21} v_m + S_{22} f_m \tag{12}$$

To show the existence of matrices S_{ij} that satisfy Eqs. (11) and (12), it is necessary to examine the resulting set of $N_p N_m + N_p M + M^2 + N_m M$ linear equations, which have the same number of unknowns. In particular, Broussard has shown the existence of a unique solution provided that the matrix

$$E = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \tag{13}$$

is invertible and provided that a mild eigenvalue condition, which is almost always satisfied, can be established.[5] However, if Eq. (13) does not have an inverse, then it is possible to have either no solutions or an infinity of bounded solutions.

Thus, for subsequent analysis, we define a new error

$$e = v^* - v \tag{14}$$

and seek a controller which guarantees that the error e approaches zero as time t approaches infinity. When $e = 0$, i.e., when $v^* = v$, we have $Cv = Cv^* = C_m v_m$, which is the result that is required.

It should also be noted here that even though the subsequent CGT-based analysis is valid only when f_m is a step command, more general time variable inputs can be accommodated by incorporating the results of Refs. 6 and 7.

2. Adaptive Control Algorithm

The resulting adaptive control algorithm is of the form[1]:

$$f = K_e(y_m - y) + K_v v_m + K_f f_m = K_r r \tag{15}$$

where

$$r = \begin{bmatrix} y_m - y \\ v_m \\ f_m \end{bmatrix}$$

$$\dim r = N_r = M + N_m + M$$

and where the gains are such that

$$K_r = K_I + K_p \tag{16}$$

$$K_p = (y_m - y) r^T \bar{T} \tag{17}$$

$$\dot{K}_I = (y_m - y) r^T T \tag{18}$$

T and $\bar{T}$ are matrices to be specified by the designer in accordance with the following stability analysis.

As shown in Ref. 1, stability of this system can be established by using the Lyapunov function

$$V(e, K_I) = e^T(t) P e(t) + TR[S(K_I - \tilde{K}) T^{-1} (K_I - \tilde{K})^T S^T] \tag{19}$$

where P is an $N_p \times N_p$ positive definite symmetric matrix, $\tilde{K}$ is an $M \times N_r$ matrix (unspecified) $= (\tilde{K}_e \tilde{K}_v \tilde{K}_r)$, S is an $M \times M$ nonsingular matrix, and T is an $(N_r \times N_r)$ nonsingular positive definite matrix, i.e.,

$$T > 0 \tag{20}$$

and e is the error $[v^*(t) - v(t)]$, which satisfies

$$\begin{aligned} \dot{e}(t) &= \dot{v}^*(t) - \dot{v}(t) = A[v^*(t) - v(t)] + B[f^*(t) - f(t)] \\ &= Ae(t) + B[S_{21} v_m + S_{22} f_m - K_r r] = (A + BK_e C_p)e \\ &\quad + B(S_{21} - K_v) v_m + B(S_{22} - K_f) f_m \end{aligned} \tag{21}$$

As shown in Ref. 1, extraction of the time derivative of Eq. (19), subject to Eq. (21) and the additional constraints that

$$C=(S^TS)^{-1}B^TP \tag{22}$$

$$\tilde{K}_v=S_{21}, \qquad \tilde{K}_f=S_{22} \tag{23}$$

yields

$$\dot{V}=e^T[P(A-B\tilde{K}_eC)+(A-B\tilde{K}_eC)^TP]e$$
$$-2e^TPB(S^TS)B_e^TPr^T\bar{T}r \tag{24}$$

A more useful form of this last relation results from incorporation of Eq. (22) into Eq. (24) as follows:

$$\dot{V}=e^T[P(A-B\tilde{K}_eC)+(A-B\tilde{K}_eC)^TP]e$$
$$-2e^TC^T(S^TS)Cer^T\bar{T}r \tag{25}$$

Two distinct possibilities are now considered, depending upon whether or not the first term on the right-hand side of Eq. (25) is negative definite or negative semidefinite.

If $\bar{T}\geq 0$, and if the first term on the right side of Eq. (25) is negative, i.e., there exists a $\tilde{K}_e$ such that

$$Q=P(A-B\tilde{K}_eC)+(A-B\tilde{K}_eC)^TP<0 \tag{26}$$

then, as previously shown in Refs. 1 and 2 with results from Ref. 8, the adaptive system will be asympotically stable. Thus, $e\rightarrow 0$ or $v\rightarrow v^*$ and, consequently, $y\rightarrow y_m$. Simultaneous satisfaction of Eq. (22) and the inequality of Eq. (26) is the equivalent to requiring that the closed-loop plant input/output transfer function

$$Z(s)=C(SI-A+B\tilde{K}_eC)^{-1}B \tag{27}$$

be strictly positive real for some gain matrix $\tilde{K}_e$, which is not needed for implementation.

However, since structural systems exhibit positive realness and not strict positive realness,[4] it is useful to reconsider the stability when the first term is negative semidefinite, i.e.,

$$Q=P(A-B\tilde{K}_eC)+(A-B\tilde{K}_eC)P\leq 0 \tag{28}$$

This along with Eq. (22) means that the transfer function $Z(s)$ is positive (not strictly) real. Since it is now possible to have

$$\dot{V}(e)=0 \quad \text{for} \quad e\neq 0$$

trajectory limit sets rather than specific equilibrium points must be considered.

To this effect, recent extensions of LaSalle's invariance principle[9] for nonautonomous systems[10,11] will be used to establish the following stability theorem:

Theorem: The output following error of Eq. (21) is asymptotically stable if

$$Q=P(A-B\tilde{K}_eC)+(A-B\tilde{K}_eC)^TP\leq 0 \tag{29}$$

$$\bar{T}\geq 0 \tag{30}$$

and if $\bar{T}$ is selected such that the second term in Eq. (25) does not vanish for $Ce\neq 0$.

Proof: If Eq. (22) is satisfied, then

$$V(e,K_I)\geq 0 \qquad \dot{V}(e)\leq 0$$

Hence, by theorem A1 (see Appendix) all solutions (e,K_I) of Eqs. (18) and (21) are bounded. By theorem A2 all solutions of Eq. (21) will therefore approach asympotically a bounded element of the set

$$\{e\,|\,\dot{V}(e)=0\} \tag{31}$$

which, by Eq. (25), is equivalent to $\{e\,|Ce=0\}$. Since this implies that the system output approaches an ideal trajectory (y^*), it implies in fact uniform asymptotic stability of Eq. (21).

The condition on $\bar{T}$ can easily be satisfied by defining

$$T=\begin{bmatrix} T_e & 0 & 0 \\ 0 & T_m & 0 \\ 0 & 0 & T_u \end{bmatrix}$$

where

$$T_e(M\times M)>0 \tag{32a}$$

$$T_m(N_m\times N_m)\geq 0 \tag{32b}$$

$$T_u(M\times M)\geq 0 \tag{32c}$$

In this case, it is not necessarily true that $v\rightarrow v^*$, if v^* is the unique solution of Broussard's equation; however, the fact that $Ce(t)\rightarrow 0$ will, from Eq. (18), imply that K_I approaches a constant value. Then, from the definition of $v^*(t)$ and f^* in Sec. II.C, the state error equation will be asymptotically stable.

III. Example: Modal Control of a Simply Supported Beam

A. System Description

In this section, we present the results of some numerical studies on the adaptive control of a simply supported beam. The beam dynamics are modeled by the Euler-Bernoulli partial differential equation

$$mu_{tt}(x,t)+EIu_{xxxx}(x,t)=F(x,t) \tag{33}$$

for $0<x<L$ and $t\leq 0$ where $u(x,t)$ is the transverse displacement of the beam and $F(x,t)$ is the applied force distribution. For conveniene we set the beam parameters m (mass per unit length), I (moment of inertia), E (modulus of elasticity), and L (length of the beam) to unity. The boundary conditions for a simple (pinned) support will be:

$$u(0,t)=u(1,t)=0 \qquad u_{xx}(0,t)=u_{xx}(1,t)=0 \tag{34}$$

If we assume a solution to Eq. (33) of the form

$$u(x,t)=\sum_{k=1}^{\infty}u_k(t)\phi_k(x) \tag{35}$$

then the mode shapes $\phi_k(x)$ will be found from the eigenvalue equation

$$(\phi_k)_{xxxx}=\lambda_k\phi_k \tag{36}$$

the solution of which is[12,13]

$$\lambda_k=(k\pi)^4 \tag{37a}$$

$$\phi_k=\sin k\pi x \tag{37b}$$

Furthermore, if the external force $f_i(x,t)$ is to be applied at points x_i, then it can be shown that[13]

$$\ddot{u}_k+\lambda_ku_k=\sum_i 2\phi_k(x_i)f_i(t) \qquad k=1,2,\ldots \tag{38}$$

Note that the solution of the original partial differential equation now requires the solution of an infinite number of ordinary differential equations.

The corresponding control problem will now be to determine the forcing terms $f_i(t)$ applied at x_i in order that some measured output behaves in a desirable fashion. To this effect, assume the existence of sensors at points z_i capable of measuring either the position $u(z_i,t)$ or velocity $\dot{u}(z_i,t)$. The corresponding observation equations then become[13]

$$u(z_i,t) \equiv Y_i(t) = \sum_{k=1}^{\infty} u_k(t)\phi_k(z_i) \tag{39a}$$

or

$$\dot{u}(z_i,t) \equiv \dot{Y}_i(t) = \sum_{k=1}^{\infty} u_k(t)\phi_k(z_i) \tag{39b}$$

In the state variable format, the above description can be rewritten in the form defined by Eqs. (3-5) by making the following definitions:

$$v_N^T = (u_1,u_2,...,u_N,\dot{u}_1,\dot{u}_2,...,\dot{u}_n)$$

$$v_R^T = (u_{N+1},u_{N+2},...,\dot{u}_{N+1},\dot{u}_{N+2},...)$$

Thus, Eq. (3) becomes

$$\dot{v}_N = \begin{bmatrix} 0 & I_N \\ -\Lambda_N & 0 \end{bmatrix} v_N + \begin{bmatrix} 0 \\ B_N^0 \end{bmatrix} f(t) \tag{40}$$

where

$$\Lambda_N = \text{diag}[\pi^4, (2\pi)^4, ..., (N\pi)^4]$$

$$i\text{th column of } B_N^0 = 2(\sin\pi x_i, \sin 2\pi x_i, ..., \sin N\pi x_i)^T$$

Similar results hold for the residual states but with infinite-dimensional matrices.

Note that for purposes of model selection, damping can be introduced into the system description by changing A_N in Eq. (40) to:

$$A_N = \begin{bmatrix} 0 & I_N \\ -\Lambda_N & -2\zeta\Lambda_N^{1/2} \end{bmatrix} \tag{41}$$

B. Model Reference Adaptive Control Applications

In order to assess the feasibility of applying the model reference adaptive control procedures discussed in Sec.II.C, it is necessary to take into account the positive real constraints discussed in Sec.II.C.2. Paralleling recent results of Benhabib et al.,[4] stability properties will be considered for the situation in which the sensors and actuators are collocated (i.e., $x_i = z_i$).

Furthermore, the facilitate the subsequent stability analyses, the complete state vector v, Eq. (6) will be redefined as:

$$v = (u_1,u_2...,u_N, u_{N+1},u_{N+2},...,\dot{u}_1,\dot{u}_2,...\dot{u}_N\dot{u}_{N+1}...,)$$

Thus

$$A = \begin{bmatrix} 0 & I \\ -\Lambda & 0 \end{bmatrix} \tag{42}$$

where

$$\Lambda = \text{diag}[\pi^4, (2\pi)^4, ..., (N\pi)^4, ...]$$

$$B = \begin{bmatrix} 0 \\ B^0 \end{bmatrix} \tag{43}$$

i column of $B^0 = 2\sin\pi x, \sin 2\pi x_i, ..., \sin(N\pi x_i)...$

The exact form of the output matrix C is dependent upon the type of sensors that are being incorporated. If only positional output is available

$$C = [\tfrac{1}{2}(B^0)^T \mid 0] = [C^0 \mid 0] \tag{44}$$

If only velocity output is available

$$C = [0 \mid \tfrac{1}{2}(B^0)^T] = [0 \mid C^0] \tag{45}$$

Finally, if a combination of position and velocity is available at each sensor location, then

$$C = \tfrac{1}{2}[\alpha(B^0)^T \mid (B^0)^T] = [\alpha C^0 \mid C^0] \tag{46}$$

Stability under each of these output configurations will now be addressed.

1. Position Measurements Only

In this case, since there is no way to introduce damping, it is not possible to even find a stabilizing feedback gain $\tilde{K}_e$. Thus, output model following is impossible by either conventional control or adaptive control [since Eq. (27) will not be positive real].

2. Velocity Measurements Only

For velocity measurements, the output matrix C is of the form defined in Eq. (45). Examination of the matrix E [Eq. (13)], in this case, indicates that inversion is not possible, and hence there is no unique set of matrices S_{ij} satisfying Eqs. (11) and (12). However, further analysis indicated that there exists an infinite set of possible S_{ij} solutions if the reference model is defined by the matrices:

$$A_m = \begin{bmatrix} 0 & I \\ -\Lambda_{N_m} & -2\zeta\Lambda_m \end{bmatrix} \tag{47}$$

$$B_m = \begin{bmatrix} 0 \\ B_m^0 \end{bmatrix} \tag{48}$$

$$C_m = [\ 0 \mid C_m^0\ ] \tag{49}$$

In a sense this corresponds to a beam with specified values for both the damping and natural frequencies. It should be noted that if an arbitrary model structure [unlike Eqs. (47-49)] is specified, then it is possible that no valid CGT solution exists.

Toward the goal of establishing the satisfaction of the positive real constraint, select P [as defined in Eq. (28)] as

$$P = \begin{bmatrix} \Lambda & 0 \\ 0 & I \end{bmatrix} > 0 \tag{50}$$

and $\tilde{K}_e$ as any positive definite symmetric matrix. Then Eq. (22) will be satisfied for $(S^TS)^{-1} = I$. Evaluation of Eq. (26) then gives

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & C^T\tilde{K}_eC \end{bmatrix} \geq 0 \tag{51}$$

Thus, if $\bar{T}$ is selected in accordance with Eq. (32), $C^0\dot{e} \rightarrow 0$ or $\dot{y} \rightarrow \dot{y}_m$. This result implies that if only velocity sensors are avaialble, the plant's velocity outputs will follow the corresponding model's velocity outputs; thus, position following will not necessarily be possible, as will be illustrated in Sec. IV.

3. Combined Position and Velocity Output

It the output is equal to the velocity plus a scalar (α) times the position, then the output matrix C will be of the form given by Eq. (46). In this case, the matrix E [Eq. (27)] will be nonsingular, and thus a unique solution to the CGT problem will exist. Toward the establishment of the positive real constraint, we consider the represenation [Eq. (41)] of the plant, with $\zeta \neq 0$. Select

$$P = \frac{1}{2}\begin{bmatrix} \Lambda & \alpha I \\ \alpha I & I \end{bmatrix} \tag{52}$$

and

$$\tilde{K}_e = K_I^T K_I \tag{53}$$

With these selections Eq. (22) will be satisfied for $(S^T S)^{-1} = I$ and P will be positive definite for

$$\alpha^2 < \pi^4 = \min(\lambda_i) = \lambda^* \tag{54}$$

This latter condition implies that the allowable percentage of positional interaction in the output is limited. Finally, computation of Q from Eq. (26) gives

$$Q = \alpha \begin{bmatrix} (\Lambda^{1/2} - \alpha\zeta I)^{1/2}\Lambda^{1/4} & \sqrt{\alpha\zeta}\Lambda^{1/4} \\ 0 & \sqrt{\frac{\zeta}{\alpha}}\Lambda^{1/4} \end{bmatrix}$$

$$\times \begin{bmatrix} (\Lambda^{1/2} - \alpha\zeta I)^{1/2}\Lambda^{1/4} & 0 \\ \sqrt{\alpha\zeta}\Lambda^{1/4} & \sqrt{\frac{\zeta}{\alpha}}\Lambda^{1/4} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \zeta\Lambda^{1/2} - \alpha I \end{bmatrix}$$

$$+2\begin{bmatrix} B_0 & 0 \\ 0 & B_0 \end{bmatrix}\begin{bmatrix} \alpha K_I^T \\ K_I^T \end{bmatrix}[\alpha K_I \; K_I]\begin{bmatrix} (B_0)^T & 0 \\ 0 & (B_0)^T \end{bmatrix} \tag{55}$$

From Eq. (55) it can be seen that $Q \geq 0$ provided that

$$\alpha \leq \min(\zeta\sqrt{\lambda^*}, \quad \zeta^{-1}\sqrt{\lambda^*}) \tag{56}$$

Finally, substitution of Eq. (55) into Eq. (25) shows that $\dot{V} \to 0$ implies

$$C^0 e \to 0 \tag{57}$$

$$C^0 \dot{e} \to 0 \tag{58}$$

Thus, $y \to y_m$ and $\dot{y} \to \dot{y}_m$.

IV. Simulation Results

A. Sixth-Order Regulator

To illustrate the regulator capabilities of the adaptive control algorithm, a sixth-order beam representation was forced to follow the trajetory of a fourth-order reference model. Thus, the dimension of the nominal state vector v_N was 4, while the dimension of the residual vector v_R was 2. The beam damping factor ζ, shown in A_N of Eq. (41) was 0.01 while the model damping factor was 0.7. The model's natural frequencies were the same as the first two beam modes, i.e., $\omega_n^2 = \pi^4$, $(2\pi)^4$. All nominal position states were initially set to 1.0, while all nominal velocity states and all residual states were intially set to zero. Since a regulatory response was of interest, $u_m = 0$. Also, all initial gains were set to zero.

First considered was the case in which both the force actuator and a velocity sensor were located at 1/6, i.e., $x_I = z_I = 1/6$. Results shown in Fig. 1 (open loop) and Fig. 2 (adaptive) reveal that adaptive control based upon velocity feedback does indeed stabilize all nominal and residual states.

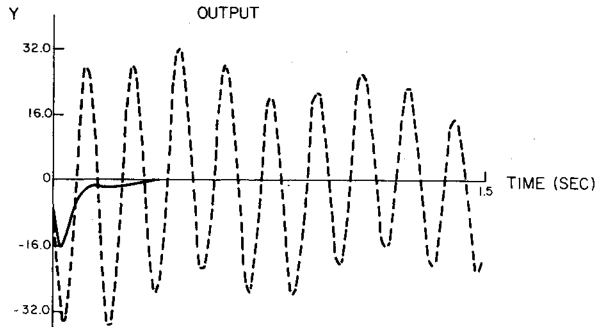

Fig. 1 Model (solid line) and plant outputs for sixth-order regulator, $z_I = x_I = 1/6$, no adaptation.

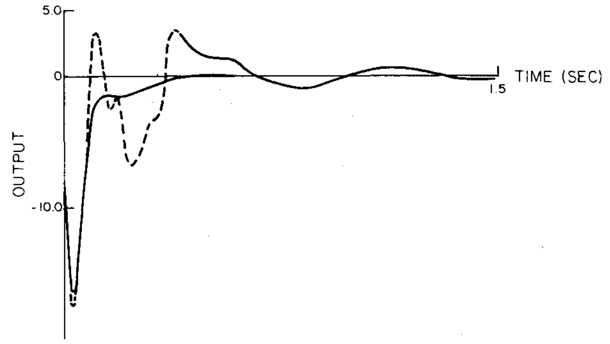

Fig. 2 Model (solid line) and plant outputs for sixth-order regulator, $z_I = x_I = 1/6$, adaptive $T = \bar{T} = I$.

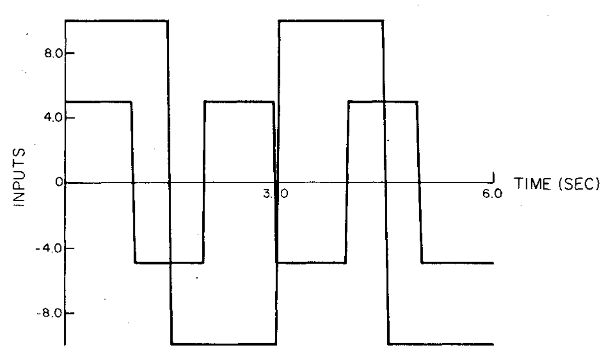

Fig. 3 Command inputs for eighth-order servo problem.

Additional studies with this representation revealed the following:

1) A magnitude increase in the matrices T and $\bar{T}$ [Eqs. (17) and (18)] resulted in a response with a smaller settling time.

2) Results for $z_I = x_I = 5/6$ were comparable to those from the previous case (i.e., $z_I = x_I = 1/6$).

3) Adaptive control was not satisfactory when only position feedback was incorporated.

4) Weighted combinations of position and velocity feedback gave responses comparable to those in Fig. 2 when $z_I = x_I = 1/6$.

B. Eighth-Order Servo Following Example

In order to demonstrate the feasibility of using the model reference adaptive algorithm for set point positioning, an undamped eighth-order representation with modes $(\pi)^4$, $(2\pi)^4$, $(3\pi)^4$, $(4\pi)^4$ was considered. The actuators and sensors were both located at $x_I = 0.29$ and $x_2 = 0.84$. The reference trajectories were defined by a fourth-order model containing the first two plant modes, but with damping ratios of 0.80, and square wave commands as shown in Fig. 3.

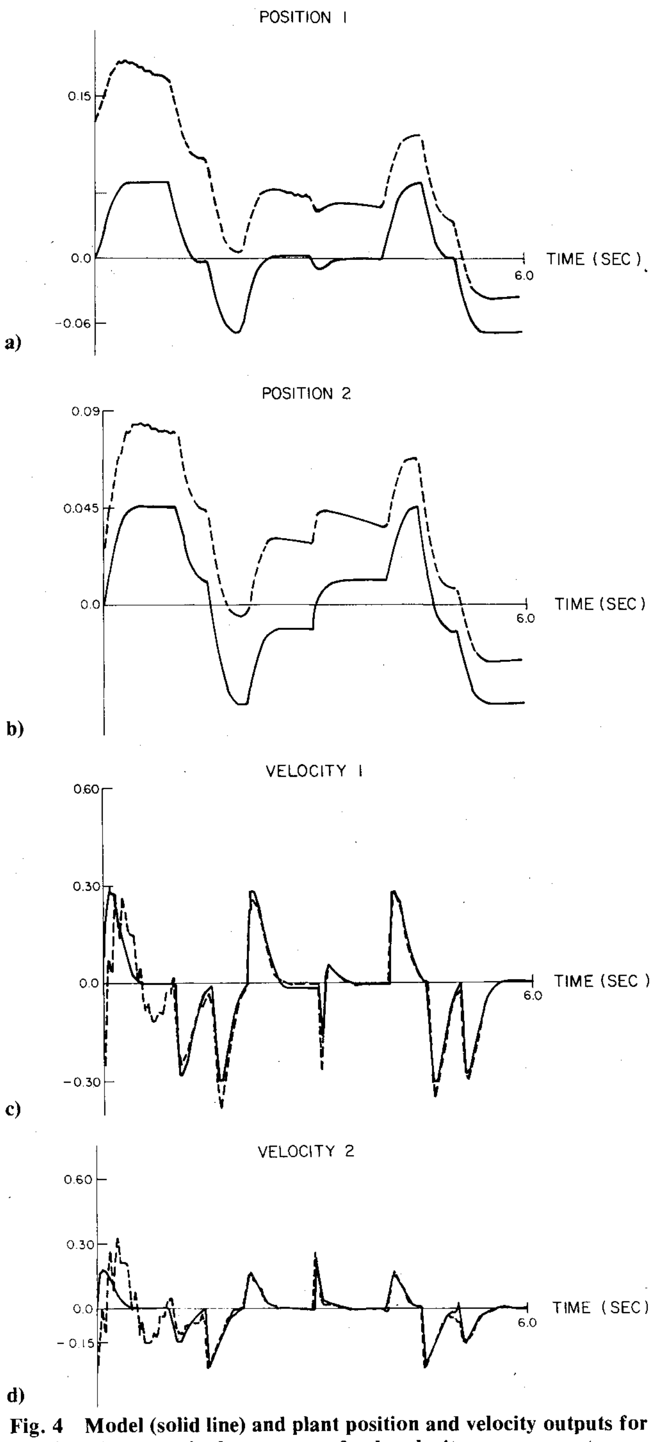

Fig. 4 Model (solid line) and plant position and velocity outputs for eighth-order servo, in the presence of only velocity measurements.

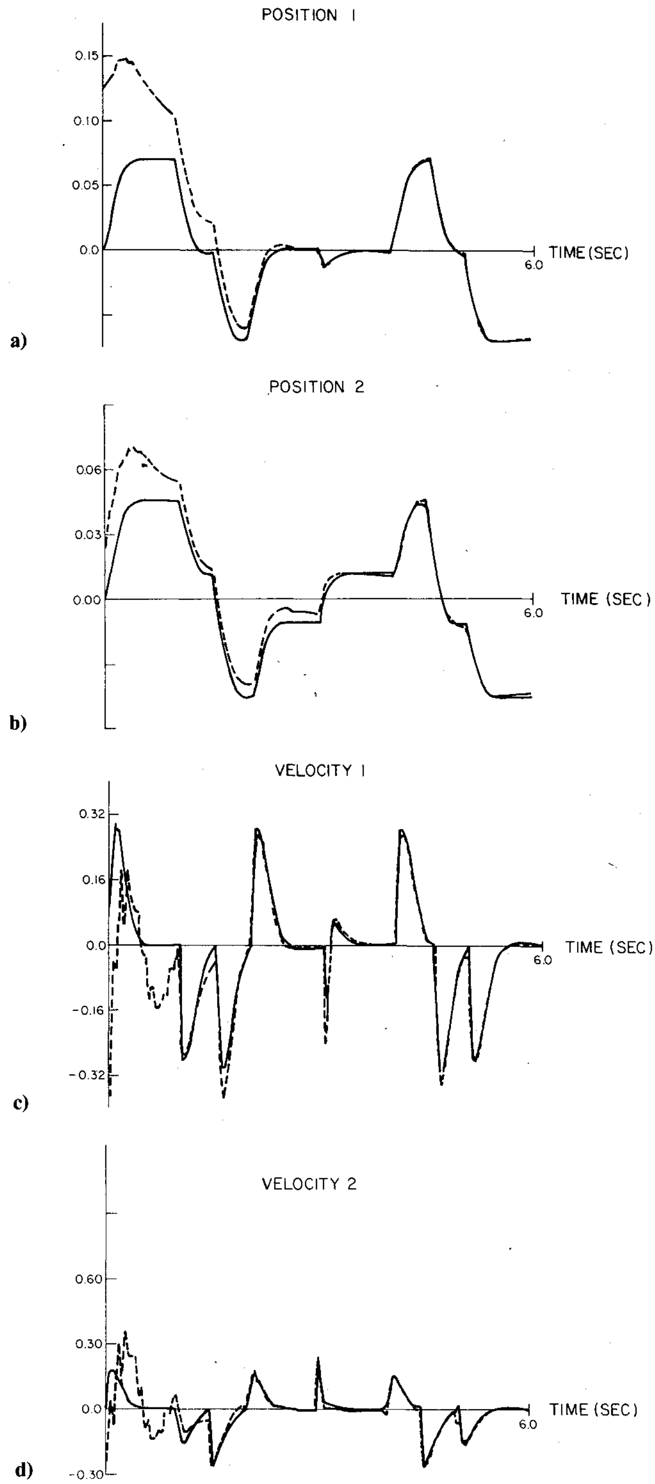

Fig. 5 Model (solid line) and plant position and velocity outputs for eighth-order servo, in the presence of combined position plus velocity measurements.

Outputs consisted either of the velocity or the combined position plus velocity at each of the two sensor locations. Results shown in Figs. 3-5 indicated that

1) Velocity outputs result only in asymptotic velocity following but not in position following. However, position errors do tend to decrease with amplitude switching of the commands.

2) Combined position and velocity outputs do yield asymptotic tracking in both position and velocity individually.

V. Conclusions and Recommendations

The feasibility of adaptively controlling large-scale structural systems without explicit parameter identification has theoretically and experimentally been analyzed using a simply supported beam. Results show that even though only positive realness (and not strict positive realness) is satisfied, output model following is achievable provided that velocity sensors are incorporated and collocated with the actuators.

The adaptive algorithm is of interest since it does not require knowledge of the structural parameters or of the system order. To widen its applicability, subsequent research should be directed toward the incorporation of noncollocated actuators and sensors and the inclusion of realistic sensor and actuator dynamics. Consideration of these factors would not only be of practical importance but also of theoretical interest since the resulting dynamics would no longer be positive real.

Appendix A: Stability Background for Nonautonomous Systems

The following results and theorems relate to quadratic Lyapunov functions which are used in Sec. II. to provide stability to the adaptation algorithms. As shown in Sec. II, the error in Eq. (21) satisfies a nonlinear, nonautononomous differential equation of the form:

$$\dot{x}=f(x,t) \tag{A1}$$

where $f(x,t)$ is continuously differentiable for all x and all $t>0$. For the material in this Appendix, it will be assumed that $f(x,t)$ is such that

$$\left| \int_{\alpha}^{\beta} f[x(\tau),\tau]\,\mathrm{d}\tau \right| < \mu(\beta-\alpha) \tag{A2}$$

where $\mu(p)$ is a nondecreasing function continuous at $p=0$ and such that $\mu(0)=0$. This assumption implies that the system of Eq. (A1) cannot move an infinite distance in a finite time.

For the sequel $V(x)$ is defined to be a quadratic Lyapunov function satisfying

$$V[x(t)]=x^T(t)Mx(t)\geq 0 \tag{A3}$$

and

$$V[x(t)]=0 \quad <=> \quad x=0 \tag{A4}$$

for some positive definite symmetric matrix M.

Define

$$G_i \triangleq \{x \mid V(x)\leq V_i\} \tag{A5}$$

Then it is readily seen that the sets G are closed and bounded and that

$$V_i\leq V_j \langle = \rangle G_i \subseteq G_j \tag{A6}$$

Theorem A1 (Bounded Solutions)

Assume that Eq. (A2) holds and also that

$$V(x,t)=\frac{\mathrm{d}V(x)}{\mathrm{d}x}f(x,t)\leq 0 \quad \text{for all} \quad x \quad \text{and all} \quad t>0$$

Then, the system of Eq. (A1) is "Lagrange stable," i.e., any bounded solution at time $t_0\geq 0$ stays bounded thereafter.

Proof: Define

$$G_0 \triangleq \{x \mid V(x)\leq V[x(t_0)]\}$$

Since $\dot{V}[x(t)]$ is continuous and $x(t_0)$ is bounded

$$V[x(t_1)]=V[x(t_0)]+\int_{t_0}^{t_1}\dot{V}[x(\tau),\tau]\,\mathrm{d}\tau\leq V[x(t_0)]$$

Let

$$G_1=\{x \mid V[x(t)]\leq V[x(t_1)]\}$$

Then $G_1\subseteq G_0$ and $x(t_1)\epsilon G_1\Rightarrow x(t_1)\epsilon G_0$. Therefore, $x(t)$ is bounded for all $t\geq t_0$.

Theorem A2 (The Invariance Principle)[10,11]

If Eqs. (A2) and (A3) hold and if

$$\dot{V}(x,t)\leq W(x)\leq 0$$

then any bounded solution of Eq. (A1) will approach asymptotically a component of the maximal invariant set of

$$\Omega=\{x \mid W(x)=0\}\cap G_0 \tag{A7}$$

where

$$G_0\triangleq\{x \mid W(x)\leq V[x(t_0)]\} \tag{A8}$$

Proof: See Refs. 10 and 11.

Theorem A3 (The Limiting Equation Theorem)[11,14]

Let

$$f(x,t)=g(x,t)+h(x,t) \tag{A9}$$

where

$$\lim_{t\to\infty} h(x,t)=0 \tag{A10}$$

Consider the limiting equation

$$\dot{x}(t)=g(x,t) \tag{A11}$$

If the limiting set Ω_c in Eq. (A7) is bounded and is asymptotically approached by the solution to Eq. (A11), then it is also asymptotially approached by the bounded solution to Eq. (A9).

In other words, the asymptotic stability status of Eq. (A11) under the conditions of Eq. (A10) determines the asymptotic stability properties of the bounded solutions of Eq. (A9).

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